Unitary Tensor Categories as Generalized Symmetries

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- **UTCs generalize groups and their representations, acting as quantum symmetries on non-commutative spaces.**
- By Popa's Reconstruction Theorem, every UTC $\mathcal{C}$ acts on some W*-algebra M by $\mathbb{G} : \mathcal{C} \xrightarrow{\otimes} \mathrm{Bim}(M)$. **[BHP13]**

By adapting subfactors techniques, we obtain that UTCs also act on C*-algebras!

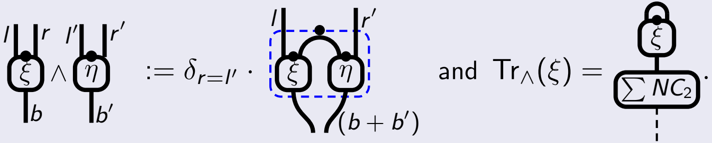
Theorem: arXiv:2005.09821

[Hartglass, HP]: Every UTC acts on some unital separable monotracial GJS C*-algebra B; i.e. there exists a fully-faithful action

$$\mathbb{F} : \mathcal{C} \xrightarrow{\otimes} \mathrm{Bim}(B).$$

A Guionnet-Jones-Shlyakhtenko C*-algebra B from a UTC:

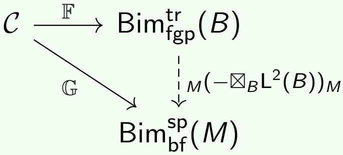
Construct algebra $\bigoplus_{b,l,r\geq 0} \mathcal{C}(x^{\otimes b} \to x^{\otimes l} \otimes x^{\otimes r})$, with diagrammatic operations:



♣ Using these tools yields a fully-faithful $\mathbb{F} : \mathcal{C} \xrightarrow{\otimes} \mathrm{Bim}^{\mathrm{tr}}_{\mathrm{fgp}}(B)$.

Hilbertifying C*-bimodules:

Extending to W*-algebras gives a monoidal faithful functor, which recovers action constructed by Brothier-Hartglass-Penneys.



Perspectives

Contrasting Remarks: W* v.s. C*

- The II_1-factor $L\mathbb{F}_\infty$ is a universal receptacle for UTC actions.
- K-theoretic obstructions: No such universal C*-algebra.

Corollary of GJS-Construction:

Every *Unitary Fusion Category* acts on the same GJS C*-algebra.

Some future research directions:

♣ Describe quantum symmetries of classifiable C*-algebras.

♣♣ In the spirit of subfactors: Can we study inclusions of C*-algebras using categorical data?

Thank you!